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Philosophy of Education

The use of Constructivist methods in teaching mathematics

“How Would Elmer Fudd Teach Math?”

Elmer Fudd is a hunter. Most ADD (Attention Deficit Disorder) and ADHD (Attention Deficit Hyperactivity Disorder) diagnosed children are hunters, as defined by Thom Hartman in his series of books “Hunter in a Farmer’s World.” These children tend to change topics and interests often, moving their focus of attention, sometimes boisterously, from one activity to another rapidly. Standard/traditional teaching techniques, such as lecture, and rote memorization, have been shown not to suit these types of learners well. This paper will argue, on both pragmatic and philosophical grounds, in favor of the combining of constructivist teaching methods with traditional methods.

Pragmatically, a person who is able to gain and use knowledge on his own, examining ideas critically and taking initiative, will be a more productive member of society, and more useful, in general. Philosophically, every person has the right and responsibility to take initiative both to care for herself, and also to contribute to the collective thoughts of society. In order to pursue either responsibility or freedom, knowledge of the available options, and how to increase those options, is necessary. John Dewey, in his essay on “The Child and the Curriculum” decried the evils of dumbing down material for all children, leading to dull-brained thinking, and passivity. Both hunters and farmers, to be responsible for their own lives, must be able to take initiative, think critically, and apply newly learned information. Traditional teaching is being shown to fall short with the vast majority of students in this regard, as well.

Constructivism, which can be defined as the forming of a mental model in response to being placed in an environment that stimulates active wondering, is a useful alternative to the traditional style of education which also answers both of these objections. Note that the use of constructivist techniques is meant to be in addition to, not instead of the standard teaching methods. One suggestion is to devote two or three days per week to constructivist style teaching, with the remaining days devoted to standard lecture methods. Since all are generally familiar with the traditional style of teaching, usually defined by lectures, recitations, and memorization, little time will be spent on descriptions of that teaching format. Lecture will, however, be defined as that style of instruction in which the lecturer disseminates information, orally or also in written format, either via handouts, or by writing on a board or overhead. This style of teaching will be defined, further, as the dissemination of information in verbal, written, or both formats, without interruptions or intermittent questions, or when all questions are saved for after the instructor has completed with giving out the bulk of information to the class. To summarize, lecture is defined here as the push of information from instructor to learner without substantial breaks during the lecture for questions, exchanges of information, or class participation.

If, as social reproduction theorists agree, education is a primary element in perpetuating and creating the type of society we will have in the future, it is incumbent upon us to ensure that all of the talent available in our society is developed to the fullest. Education is the vehicle that will take us there. We are obligated to create a society in which all are truly free to participate, and this is only possible when all members of society are fully trained in critical thinking. Whether we are born with all knowledge, as Socrates believed, or must learn it afresh, questioning and initiative are

crucial parts of participation in any free society. John Dewey, in his treatise “Democracy and Education,” pointed out that in order to truly learn something, the learner must absorb an idea, and take ownership of it. These concepts: ownership of an idea, putting information in context, and providing thought-provoking educational experiences, are at the heart of Dewey’s writings, and of the constructivist movement. Only by asking “why, and how, and from where,” can the learner fully internalize a piece of Information. He also felt that learning a particular subject in isolation from its context and the surrounding applications is not a complete way of learning the subject. This is in direct opposition to the traditional method of teaching each course as a subject unto itself. Geometry, as one example, is taught in complete isolation from other courses, and removed from its context. When geometry is taught in conjunction with art, or other applications, student understanding is enhanced. This context is, in fact, one part of how a teacher must, according to Dewey, provide learning experiences that encourage questioning, observation, and wondering, which leads to more thought, surrounding the subject to be learned. So how, then, does a mathematics teacher provide contextual and concrete experiences, when faced with such abstract topics as linear algebra, and matrix equations?

How would Elmer Fudd, our hunter par excellence, teach them?

Acting is a powerful teaching tool, particularly for learners who learn by moving around and using their bodies. Charlotte Perkins Gilman, in the novel *Herland*, advocated movement and play as the most effective means of learning. Acting is play at its best, allowing both the actor and the audience to engage an idea actively, both consciously considering the idea, and subconsciously, through the artistic side of the brain, simultaneously. One application of Howard Gardner’s theory of multiple intelligences involves acting out, or becoming an equation.

Mr. Fudd would probably use this technique to teach young hunters how to determine the trajectory of a bullet aimed for a rabbit, during rabbit season.

Given the equation ‘ $X + 3 = 5$ ’, two students stand for the variable X , another student for the plus sign, four other students each stand for the numbers one through four, and a student forms the equals sign, standing opposite the plus sign student. Five other students, each standing opposite a ‘number’ student, represent the numbers one through five. The evenly matched pairs of students show that the equation has been correctly solved. There are many possible variations on this theme, leaving out the plus and equal signs, or the variable, for a more clear solution of the equation, or if fewer students wish to participate.

Other uses of acting involve allowing one particularly gifted student to demonstrate a technique or concept, by becoming the concept. For example, an especially rambunctious pupil was having difficulty in one of my high school mathematics classes with the concept of reciprocals. After explaining the idea of inverse fractions several times, I asked him to do a handstand.

To the delight of both the demonstrating student and the rest of the class, the concept became much clearer as I pointed to the inverted student, and explained that we were to do the exact same thing with our fraction!

There are, of course, down sides to the use of acting as a teaching technique. One rather pointed example is the use of my overly athletic student to illustrate the concept of reciprocals. When I asked him to stand right side up, after completing my explanation of reciprocals, he promptly fell

over, landing with a crash on the floor. While Elmer Fudd might have approved, the guidance counselor in the office next to my classroom did not. Although the student was not injured in his fall, the noise certainly did create a distraction, both for my class and for others in nearby rooms. This leads us to another pitfall of acting as a teaching technique. Acting can often be a noisy and fast-paced. It is not easy to maintain proper teaching decorum over a classroom full of students, whether children or adults, even, when somewhere else in the classroom, one or more individual students are moving around, making noise, or even standing silently in a distracting pose –on one's head, for example. There must be a focus on the idea to be learned, in order for the experience of acting to be of educational benefit, and that focus can easily be lost in the hustle and bustle of a group of actors showing off in front of a crowd. An additional concern with acting is that it does require imagination.

Not everyone will benefit from acting out or watching the portrayal of a concept, since not everyone learns through movement or body language. Acting may thus be a waste of time for non-kinesthetic based learners. While they may enjoy the show as a form of entertainment, which is arguable valuable for education in itself, these students will miss the point of the actual lesson, unless non-acting based methods are employed, in addition to acting, to illustrate the concept being taught.

Elmer Fudd would undoubtedly use acting at least occasionally, as one of the tools in his armory of young hunter training techniques. Beyond being enjoyable for restless young hunters, who are constantly on the lookout for rabbits and ducks to capture, acting as a teaching method can enhance the learning pleasure and effectiveness for young farmers as well. Mr. Fudd would be certain to remind all of the students to “be vewy verwy quiet,” and to be respectful of classmates in the entire building. To ensure that the point of the lesson is addressed in the skit, he would also be likely to give a short synopsis of the concept being illuminated by the skit, either before or after the performance. In addition to illustrating the pure mathematical concept under discussion, a skit can unobtrusively tie in the context, historical, social, or scientific, for which the math was developed. A group of students working on units of measure may take the opportunity of Patriot's Day to enact a short skit on the Battle of Marathon, “running” the distance in miles, meters, and even cubits. This brings not only context, but passion and creativity into the classroom: two things that Jonathan Mooney and David Cole, co-authors *Learning Outside the Lines*,” point to as essentials for learning, and for life itself. Acting also provides a perfect methodology for team teaching. Teaming up with one or more teachers to combine several classes for a short time, with a specific purpose defined can work nicely, if planned out well beforehand. As pointed out by TheodoreSizer in the first book of his “Horace Trilogy,” *The Dilemma of The American High School*, team teaching can cause confusion and even be counterproductive, if a central focus and teacher coordination are not maintained. As an example, several students for a class that is studying arachnids in science, and Cartesian coordinates in math, can act out the myth of Arachne's contest with Athena.

A history or social studies class could even join in, if enough room is available. Each student can take turns at the loom, and keep samples of the weaving. The geography, language, attitudes, and clothing of ancient Greece can be taught through this skit, as well as the grid coordinate system, of course, using a real cloth example. Latitude and longitude lines can be compared to the X and Y axis, referring to the warp and weft that the students created with their own hands. Not to mention the unfortunate Arachnid.

Another well-respected constructivist technique that Elmer Fudd would likely have occasion to use is that of building things. It is generally acknowledged that if one is able to build a working item, of almost any kind, then that individual has mastered the principles involved in its making. While this

may sometimes be up for debate, it is undeniable that to build a thing is to involve some practical application of at least a few concepts. Practical application is often the best way to understand a concept, and also gives the satisfaction of having produced a tangible object when completed. Vocational schools are often popular for this very reason—they allow students the opportunity to see results built by their own hands very soon. The shorter time frame between learning concepts and putting those concepts to use can be a great help and motivator for a young person (or an adult) who is apt to ask “why are we learning this?”

TheodoreSizer, in his chapter on agreement (between teacher and those taught) in Horace’s *Compromise: The Dilemma of The American High School*, argues that sometimes letting students discuss what interests them, and then pointing out the curricular application in that topic, can be more effective than doggedly sticking to the prepared lecture. If that happens to be building an electronic circuit, as it was in my Algebra1A class, one day, then building a hands-on model for display can be more instructive than any textbook work, or lecture. As it happened, on this particular day, we were actually reviewing graphs and charts. A student interrupted my lecture to comment about his heartbeat, so I took the opportunity to return to the topic of the day by explaining how to graph a heartbeat in terms of beats per minute. I then asked the class to draw a series of graphs, from flatliners to 70 beats per minute. The gregarious student, stymied that I had redirected his comment, began to talk about his electronics project with several of his classmates. I used this conversation as an opportunity to review the solution of single step equations, using Ohm’s Law as a starting point. At least for that particular student, this proved to be more interesting, and he came back after class for several days in a row to work out the equations needed to determine how to build his circuit.

Rousseau and Elmer Fudd would very likely agree on one thing –Emile, like Mr. Fudd’s students, will learn best by doing, and experimenting, and building. From tree-stands to bows, arrows and quivers, and maybe even muskets and balls, young hunters under either of these two hands-on teachers would learn by doing and building. Even practical application has its down sides. Take the case of a hands-on map-building project, described on the world wide web site <A href=”<http://www.nsta.org/programs/laptop/lessons/h2.htm>”><http://www.nsta.org/programs/laptop/lessons/h2.htm>

which points to several pages displaying constructivist lesson plans for mathematics education. This project used laptops and GPS (Global Positioning System) units. The project set before students the objective of mapping a park and its surroundings on a student-created topographical map. Students discussed and were taught the general elements of cartography, then provided equipment and one adult guide for each group of students, and encouraged to discover for themselves the challenges of mapping out an area. This is a wonderful idea, but how many school districts will realistically be able to implement such a project, given the expense of a laptop, GPS unit, and even a simple topographical map? Any one of these items may be beyond the reach of a school district, particularly in an inner city struggling for basic funding of any kind.

Even in cases where money is not an issue, many schools face the problem of limited space. At least one Middle-High school in New Hampshire uses trailers for temporary classroom space, and even shares space with a neighboring school. Given constraints like these, it may be difficult to find the room needed to spread out enough to build individual projects, store them, or even manage to transport them through the halls, crowded as they generally are. Floor space, table space, storage when not being worked on, even the extra bodies needed to move large projects can be a hindrance to hands-on projects in the classroom.

Where money and space are no problems, one last great objection can be made: time. It certainly can take much more time to build a model pyramid in a geometry class than to simply explain the angles involved. Measuring its angles, determining a proper size for the base, lining up each side of the pyramid on top of the base, fitting them all together, and cleaning up the mess when done, all takes more time than a lecture. Explaining each step of the process, before, during, and after the work of building the structures adds to that expenditure. This does not count the time involved in obtaining the materials to be used in building the project, or even the time spent in determining what materials are to be used for the project. All of these tasks take away from time needed to cover the required material.

While it is important to cover all of the required material, it is equally, if not more important to help learners absorb what is being covered. Rousseau would have argued that less is better, and that anything covered must always be done through building. As with his example of Robinson Crusoe, whatever is taught must be taught through experience and practical experimentation. Elmer Fudd might have to remind Emile, though, that the consequences of firing a musket improperly could be rather permanent, and so, learning to read is a necessity in order to avoid fatal experimentation. Thus, not everything is best taught by hands-on methods. Reading the directions can be both more efficient, and even life-saving. Keeping that in mind, Elmer Fudd would have to balance the impatience of young hunters against the cautiousness of young farmers. Mr. Fudd would also remember to balance the need to inspire passion in both groups against the need to cover all of the requisite mathematics to be able to count the number of days from duck season to rabbit season. That is required by the standard curriculum guide for all of the mathematics classes. One possible solution would be to set up a schedule ahead of time displaying the start of both seasons, with a few planned hunting expeditions for the classes. Most days, Mr. Fudd would likely cover the standard math, using lecture format. He could then periodically remind his students that once they have learned enough of the required math, they would be able to more effectively go on their planned hunting expeditions. In the meantime, as an optional homework project, individual students could be allowed to research and build model rabbits or ducks to show off to their classmates, and explain the various uses rabbit and duck parts could be put to after their expeditions. Mr. Fudd would always make sure to point out the various mathematical topics and principles that were used in the creation of these models, and tie them into the ongoing classwork. That would give the students a context into which to put both the previous, current, and upcoming classwork and homework. He would also allow the students to help planning the expeditions, which would keep all of the students engaged in and looking forward to both the upcoming trips, and the ongoing classwork which is in preparation for those trips. That way a smaller number of projects could be stretched across more lecture format classes, while holding the attention of the young hunters in the classroom.

Two teachers at Nashua Senior High School, a tenth through twelfth grade public school in Nashua, NH, have found less glamorous, but equally useful hands-on applications for their students. An algebra teacher assigns a project allowing her students to use various equations to create a fold-out fan, decorate the fan to taste. The other teacher, teaching an advanced placement calculus course, shows the application of integrals and the area under the curve by assigning a wine-glass project, in which more complex equations are applied to the building of a wine-glass out of available materials. Each of these teachers is reinforcing concepts learned with practical application, demonstrating knowledge, and allowing for personal creativity.

If Elmer Fudd were teaching basic mathematics, he would undoubtedly use a set of model rabbits. They would fill a variety of functions, from illustrating whole numbers, to holding the place of the target when demonstrating how a parabolic equation describes the trajectory of a

musket ball fired during rabbit season. These model rabbits are one example of manipulatives that can be used in the classroom to help learners construct models of concepts under discussion. Blocks are another type of manipulatives, as are geometric shapes such as trapezoids, cubes and spheres used to illustrate three dimensional modeling. Other forms of manipulatives include the Jewish custom of baking cakes in the form of various symbols to be learned. This is traditionally done to illustrate that learning should be sweet, but can also be seen from another point of view. In addition to its tasty quality, and the benefit of being able to touch and feel the symbol, which does drive it more effectively into the memory, there is an additional intangible quality. Nel Noddings, in her essay on "Caring," stresses that a teacher is first and foremost a care-giver, in the role of giving care to those being helped to learn. She also asserts that the more time and the more individual subjects spent with learners by each teacher the more that teacher will be able to help model and connect with those learners. This modeling of thought and connecting with the learner is an essential part of teaching. Nothing connects people better than the sharing of food.

A mathematical application of the sweet concept mentioned above could take the following form. Bake cakes for a class studying number theory in the shape of the symbols for infinity, existence, and, or, and not. Then, draw the symbol for Does Not Exist up on the board. Elmer Fudd would proceed to point out that, much like a rabbit during duck season, there is no cake for that particular symbol since it Does Not Exist!

Another application of the use of manipulatives in the classroom is the use of Lego and Erector Set blocks. From multiplication and division, to fractions, to set theory, blocks are an inexpensive way of providing hands-on explanations for kinesthetic and visual learners. They can also be used for short building projects that allow spatial-pictorial learners to use as three-dimensional spaces upon which other concepts can be built. For instance, as the container for the imaginary duck who will travel in a parabolic arc when falling during duck season.

Even the use of manipulatives, tasty and space-saving though they can be, has its downside. To many critics, they can often appear to be used unnecessarily. Why play with blocks when explaining a concept on the board will do? Then there is the ever-present spectre of funding. Manipulatives of any kind will certainly cost more money than simply drawing on the board would cost. Then there is the additional custodial cost of cleaning up after the class that used manipulatives, quite often. Anyone who has had the experience of being forced to pick up hundreds of tiny Lego pieces, or had to vacuum a floor full of cake crumbs can attest to the fact that manipulatives can certainly make a mess. That leads us to our major objection to the use of manipulative teaching aides: squandering of time. It is still far more efficient, in terms of material coverage, to use a lecture and memorization based teaching format than to explain a concept, pass around blocks, collect the blocks, and determine whether the concept was solidified by the use of those blocks. More material can be covered, and arguably more clearly, by simply writing and telling the information. What is handed out in lecture format can also more efficiently be tested on a written examination. Both the objective and the result of the use of those manipulative materials may be more difficult to define, and will certainly take longer to attain than by using standard techniques of lecture and multiple choice or single answer testing.

Elmer Fudd would likely resolve the issue of the use of manipulatives in the classroom by occasionally buying a rabbit shaped dog biscuit, using it to demonstrate trajectory in the solution of parabolic equations, and then feeding the biscuit to his hunting dogs. As that is not a luxury most teachers have, however, other suggestions will also be forthcoming. One possibility is to use a chess board and only one piece as a manipulative. As an example, in a geometry class that was studying various coordinate systems, there were several students who enjoyed playing chess. On a day when part of the class was continuing to work on the previous day's lesson, these particular students, who had completed the assignment, were given a two-part challenge. The first was to attempt a solution to a well-known problem in the world of chess, known as The Knight's Tour. The second part of the challenge was to explain what the Knight's Tour had to do with coordinate systems. In this way, a knight was used as the manipulative, and the chess board as a grid coordinate system. In solving the Knight's Tour, where the objective is to land the knight on each square on the chess board only one time, what the students were doing was solving a mapping problem, unbeknownst to them!

A more cerebral technique in the constructivist repertoire is the use of discussion. During a class discussion, facilitated by the instructor, learners formulate their own ideas of the concept in question. This allows them to take personal possession of the concept, and make the idea their own. This is something that Dewey has asserted is essential to the learning process, and Elmer Fudd would have to agree. In class discussion also allows Howard Gardner's interpersonal learners to work together, bouncing ideas off of one another, and fine tuning their understanding of the topic as they discuss it. Thom Hartman, in his book *Complete Guide to ADHD*, suggests a wonderful exercise for the interpersonal learners in the classroom. This technique is often called "Peer Mentoring." Even Elmer Fudd, who doesn't like talking very much, would highly approve of it. The instructor first gives a five minute summary of the lesson for the day. Even young hunters can sit still for that amount of time without getting too bored. Then, the class can break up into small groups. Those students who understood the short summary can each take a group, and teach the other students in that group. Another form of discussion, used by a senior calculus teacher at Nashua Senior High School, is partnering. His students break into pairs and work on a set of problems collaboratively. The problem sheet is signed by both students, and discussed the next class session after being graded. Still another, more conservative, perhaps, synthesis of lecture and constructivist methodology is the use of the Socratic method. While presenting ideas or information, asking learners, during each step, questions drawing on previous material, and pointing up the patterns that are forming in the new material as each new problem is presented can draw out the critical thinking skills and curiosity at once. One example is doing factoring, then finding zero roots of equations, and watching for the patterns that emerge as each equation is graphed. Students often work out for themselves that the zero roots will turn out to be the intercepts well before being told. In discovering these things for themselves through questioning, they are sharpening both their intellect and their self-esteem.

TheodoreSizer in *Horace's Compromise: The Dilemma of The American High School* strongly suggests that there must be connection among the various subjects in order for them to be taught or learned fully. In this regard, Sizer agrees with Sidney Hook, who, in his *Education for Modern Man*, points out that a broad understanding of the connections between subjects is necessary in order to develop the ability to imaginatively interpret information. This connection can be simulated, even if a subject is being taught in isolation from other subjects, via the use of classroom discussion. Interdisciplinary connections can be drawn, for instance between mathematics and the sciences, to give context and a wider sense to the individual unit being taught in any particular lesson. Applications from various fields can be discussed to shed a broader and more practical light on the lesson from many angles, which Mr. Fudd would use to keep the interest of the young hunters in the class.

Another method that Elmer Fudd might try of using discussion in his mathematics classroom is drawn from the book *The Elements of Teaching*, by James Banner Jr. and Harold Cannon. They give an example of a teacher who assigns her class to do individual reports on some topic, and also takes a topic on which to do a report, herself. By presenting her report orally along with each of her students, she sets an example, and also provides her students an opportunity to learn how to give constructive feedback. Mr. Fudd could assign various reports on how to determine the altitude of a duck in flight, or the acceleration of a rabbit, and himself present the report on how to determine the angle at which to hold the musket when aiming at the duck.

Locke felt that one ought to reason with children more and more as they grow older. Based on this, Locke would quite likely agree with Jane Martin who asserts that critical thinking is a basic subject that should be added to the traditional “3 R’s” as a foundational learning objective in education. When a student explains the reasoning behind an answer, that student also learns how to apply the same sort of reasoning to other areas. This practice in discussion and critical thinking, as Friere and Scheffler would agree, is crucial to the stability and perpetuation of a free society.

The biggest objection to the use of discussion as a teaching tool in mathematics classrooms is the response “What is there to discuss about mathematics?” Most people see mathematics as an either/or type of proposition. Either an answer is correct, or it is not correct. Many students feel that if they obtain a correct answer when working out a mathematical problem, there is no reason to show how they obtained the answers. Since many students are not accustomed to showing their work, and certainly not to explaining the logic by which they obtained the answer verbally, there is a good deal of resistance to overcome in discussing mathematical concepts.

Elmer Fudd would certainly try some of the discussion techniques mentioned earlier, such as peer mentoring, and reports for oral presentation. He would also likely request verbal explanations of the logic behind various answers. Most importantly, he would discuss whether it was duck season, or rabbit season!

Drawing seems to be a somewhat more radical application of constructivism, when applied to the teaching of mathematics. For visual students though, who must see and draw things in order to learn about them, being able to draw rabbits and ducks may prove helpful. Drawing will certainly be more fun for artsy types of students. Elmer Fudd would probably permit drawing, as long as it was of relevant lesson-appropriate content, such as the trajectory followed by a musket ball toward a rabbit.

The major question in an objector’s mind will be “How, exactly, do you draw $x + 3 = 10$?” Some possible solutions to this question follow. One is to have artistic students draw a representation of a binary search. This is a pattern in which the area or set of numbers being searched is narrowed down by half each time through the search pattern until a solution is found. This search algorithm can be used to illustrate both relationships among whole numbers, as well as the concept of infinity.

Music is another tool that Elmer Fudd would undoubtedly reach for. Who does not remember how the songs of SchoolHouse Rock taught us about bills, and the Preamble to the Constitution? It is also a well-known fact in the advertising industry that songs work to keep

information with us longer than any other form of advertising media. While it could be objected that students above a certain age will not want to be seen singing songs, particularly in school, once the stigma is overcome, music can be an effective teaching tool. In some circles, music may even be a badge of pride. The downside is that the music that emanates from one classroom may be heard as noise to another classroom taking a test at the time. And of course, due to the requirement to “be vewy quiet,” one can not sing while hunting.

A final example will serve to present a unified approach to the use of constructivist teaching methods in mathematics education. Given a lesson objective of teaching circumference and the area of a circle for a one to two week unit, Elmer Fudd could start this way. Before class begins, the following word problem is to be placed on a side chalk board:

It is rabbit season. Mr. Fudd has only one dog, Fido. Fido needs to be trained to follow a rabbit when hunting. Prior experience has taught Mr. Fudd that rabbits typically follow a figure eight pattern, when running to their rabbit holes. To ease training, and to avoid tiring out poor Fido, Mr. Fudd wants to build a fenced in area in which to let the rabbit loose, and teach Fido to follow it. To expedite recapturing that rascally rabbit when done, Mr. Fudd also wants to make a net to cover the entire area of this fenced in clearing. He has determined that the radius of his small clearing is 25 feet. Determine how much fencing and how much netting Mr. Fudd needs, and explain your answers.

As the class arrives, our genial Mr. Fudd greets various students, takes attendance, and settles the class down to begin his lecture:

“Hello, class. Our lesson for this week will cover ciwcumfewence, and the awea of a ciwcle.” On an overhead, or front facing chalk or white board, he would write:

C _ = (is defined as) the perimeter, or distance around, a circle.

D _ = diameter _ = the distance across a circle; the center chord

$$R = \frac{1}{2} D, \quad = D/2$$

At this point, Mr. Fudd could draw a picture of a circle, label a starting point, and proceed the entire distance around the circle, even providing a hypothetical distance in both meters and yards, to tie in the connection with previously learned units of measure and conversion. To help the pictorial learners, he could hold up an embroidery hoop, borrowed, no doubt, from Mrs. Hen. The wing nut on the hoop provides a convenient start and end point, illustrating well the distance around a circle, and what happens to the space inside the circle as that distance is changed, by adjusting the tightness of the wing nut.

He would also want to draw another circle, this time dividing it in half with a line to demonstrate that a diameter always bisects the circle. Having shown circumference and diameter, Mr. Fudd is now free to explain that there is a special ratio that links these two values: Pi.

$\pi = \text{the ratio of the C to D} \quad D = C/\pi$

π is a fixed constant; ≈ 3.14

If we know either C or D, we can find the other

That lecture portion of the lesson should take approximately twenty minutes to deliver. To help the visual-spatial and interpersonal learners, he could then allow another ten minutes for students to partner up and discuss, then write or draw an explanation of π to hand in for classwork credit. Since Elmer Fudd is a hunter himself, he would likely prefer to move around the room, encouraging the various groups of students as they work, rather than remaining at the front of the classroom.

Once their ten minutes of group work was done, Mr. Fudd could point out the problem that was written up on the side chalk board before the class arrived:

“Ok, how would you go about helping Mr. Fudd train Fido? Lets look at the pwobwem on the boawd, and I want you to wite down what values you need to detewmine, why you need to detewmine each value, and what units each value and both of youw answevs will be in.”

While the students are starting on that problem, which they can either finish in class, or as a homework assignment, Mr. Fudd could give them a heads up on an extra credit assignment. In collaboration with each other, the students could be allowed to put on a skit at the end of the second week. This skit would help them to solidify or review their understanding of all the material on circumference, area, and π being covered this week, and allow for more creativity on the part of the remaining learning styles, such as musical and artistic and kinesthetic students. They could be allowed to decide who among them would actors, set builders, scene painters, choreographers, and music masters. Allowing the students to organize the skit, with the stated caveat that the skit must relate to and help explain the use of π , area, and circumference, teaches them organizational skills, teamwork, and the underlying mathematical material, all at once. Discussion of both the problem and the skit could either be used to wrap up the remaining ten to fifteen minutes of the class, assuming a fifty minute class period, or held over to the next class meeting.

Lesson Handout: Circumference and Area of a Circle

Teacher = Mr. Elmer Fudd

$C =$ (is defined as) the perimeter, or distance around, a circle.

$D =$ diameter $=$ the distance across a circle; the center chord

$R = \frac{1}{2} D, \quad = D/2$

(See mosaics, courtesy Mr. Wile E. Coyote...)

$C=440\text{yd} \approx 400 \text{ meters}$

Pi is a fixed constant; $\pi \approx 3.14$: If we know either C or D, we can find the other.

This is an extra credit assignment for which you should collaborate with your classmates. You may put on a skit during class next Friday, that will serve as a review of circumference, area of a circle, and the meaning of Pi. You must decide who will be actors, who will build the set, what sort of scenery needs to be drawn, and what music to use. All of this must relate to and help explain the uses of Pi, area, and circumference as you would use them in your own lives.

Have fun, and Good

Hunting!

Constructivist teaching methods strive to supplement lecture methods by filling in the gaps that lecture leaves open, such as body-kinesthetic and interpersonal learning. Constructivist techniques also emphasize critical thinking and learning how to find and interpret information based on a broad range of connections. Matthew Miltich, in his recent article for the NEA Higher Education Journal entitled "All the Fish in the River: An Essay on Assessment," likens ideas and knowledge to fish to be caught. He defines the educator's job as that of helping the learners to learn how to catch those fish for themselves.

As asserted by TheodoreSizer in his section on teachers in Horace's *Compromise: The Dilemma of The American High School*, one needs a broad base of knowledge both to teach and to learn effectively. The fish require a broad net. As our society becomes more completely industrialized, and moves into the post-modern information age, a larger and larger percentage of our population will have to be well educated to provide a workforce that will allow our businesses to continue to function. Even from this strictly Machiavellian point of view, we can no longer allow the large numbers of our learners to slip through the educational cracks. It costs too much to import trained workers. That requires us to adopt new techniques in educating our learners to the minimum level necessary (which continues to rise, as the technological complexity and business requirements rise) to contribute to the workplace. From the more idealistic standpoint, ours is a democracy, and to be a full participant in a democratic society, one must be able to analyze and debate the issues, which requires training in critical thinking and analysis. Also required to function in a democracy, is the ability to draw connections between even pieces of information that may seem only remotely related to one another. As Jack Dewey points out in *Burned Out: A Teacher Speaks Out*, both learners and teachers must be exposed to a wide variety of topics within a subject. Good critical thinkers must also be able to draw upon and make for themselves the connections between traditionally separate concepts, much in the same way as connections must be inferred between such traditionally separate subjects as mathematics and history and science.

The connections are there, but are made unapparent by the strict division of subjects in modern schools.

While Jack Dewey may or may not be correct in arguing that cross-disciplinary certifications is the answer to the connections problem, there are certainly connections between each of the various subjects that are taught in schools, and there is certainly room for both traditional and constructivist methods in math teaching.